

Eldar Straume  
Room 1250, SB II  
e-mail : eldars@math.ntnu.no  
Tlf. : 73 59 66 83 (mobil: 99410389)

## SEMESTER PROJECT, DIPLOM , MASTERS, or PhD

IN GEOMETRY/TOPOLOGY AND ITS APPLICATIONS

2015-2016-

Geometry and topology play a fundamental role in the mathematical sciences nowadays. I would like to point out some topics for students at various levels – from semester project to PhD – for those interested in the applications as well as for those who have stronger theoretical interests related to these areas.

By topology I mean "general" topology, algebraic topology, or differential topology. The corresponding introductory courses at present at NTNU are the following:

- MA 3403 Algebraic topology I (fall)
- TMA 4190 Manifolds (spring)
- MA 3402 Analysis on manifolds (fall)
- MA 8402 Lie groups og Lie algebras (to be offered as a reading course)

By geometry we shall primarily mean differential gemometry, which is closely related to differential topology (TMA 4190, MA3402) and the geometric aspects of the theory of differential equations. More specifically, differential geometry is the mathematical discipline that uses the techniques of differential calculus, integral calculus, linear algebra and multilinear algebra to study problems expressed in a geometric framework. Major geometric structures are Riemannian geometry, symplectic geometry, and contact geometry.

Unfortunately, there is no course in differential geometry listed in the Emneoversikt, so what we use to do is to compose modules or reading courses for the interested students, depending on the actual choice of project. Lie group theory may be regarded as part of differential geometry; these groups play an important role as transformation groups or rather "symmetry groups" of various kinds in mathematics and the natural sciences. Although the course MA 8402 is listed as a course at the PhD level, it is in fact no more advanced than the other courses listed above.

Some project proposals:

Semester projects

1) Apply Riemannian or symplectic geometry to study a specific topic in mathematics or physics, for example in analysis or classical (Hamiltonian) mechanics.

2) Investigate the relationships between the 3-dimensional Lie groups  $SO(3)$ ,  $SU(2)$ ,  $SO(2, 1)$ ,  $SU(1, 1)$ ,  $SL_{\mathbb{R}}(2)$  and their Lie algebras.

3) Contact geometry in 3-space  $\mathbb{R}^3$ . Explain its historical roots, dating back to Sophus Lie in the 1870's, and the modern setting. Recommended literature is :

S. Lie and G. Scheffers, "Geometrie der Berührungstransformationen", first published 1896 in Leipzig , see Chelsea Publ. Company, 1977.

A. Arnold, "Mathematical methods of classical mechanics", second ed. Graduate Texts in Math., Springer-Verlag 1989.

#### masters or phd

1) Ricci curvature and Ricci flow. Of special interest are 3-dimensional compact Riemannian manifolds and 4-dimensional pseudo-Riemannian manifolds in general relativity theory.

2) A topic from the classical 3-body problem. Using equivariant geometry and geometric reduction.

3) Non-commutative geometry and spin networks (according to Penrose). Investigate whether the usage of spin networks could possibly simplify or extend the major results obtained (or not obtained) in the recent book

Pedro de M Rios and Eldar Straume: *Symbol Correspondences for Spin Systems*, Birkhäuser 2014.

4) Masters thesis project in geometry: A comparison of the approaches used by Bolyai and Lobachevski in their discovery of non-Euclidean geometry around 1830.